Electron-nucleus collisions at small x

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François Gelis

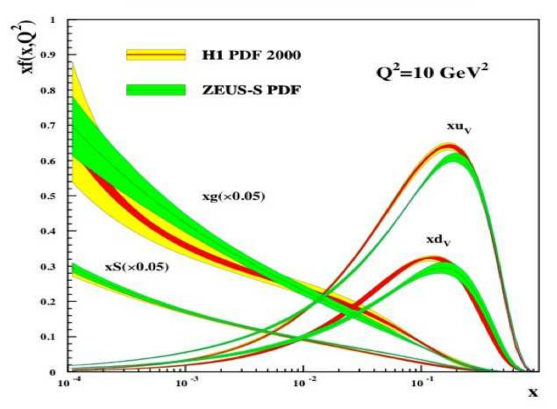
CEA, IPhT

Outline

1. **Gluon saturation at small x**
2. **Deep Inelastic Scattering**
3. **Proton-Nucleus collisions**
4. **Nucleus-Nucleus collisions**

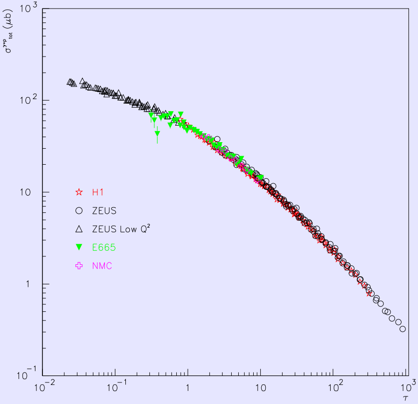
Growth of the gluon distribution at small *x*

Gluon distribution at small *x*



Geometrical scaling

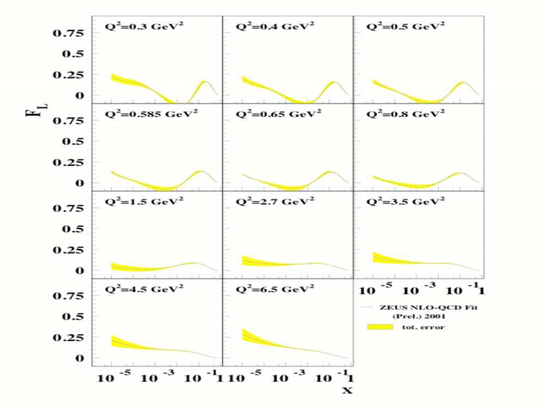
Geometrical scaling



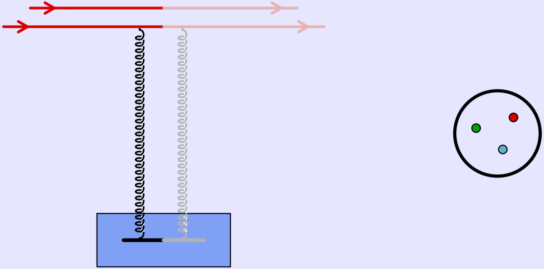
Note: geometrical scaling has also been observed for DIS on nuclear targets (E665 and NMC)

Some trouble with F_L at small Q^2

F_L from DIS fits

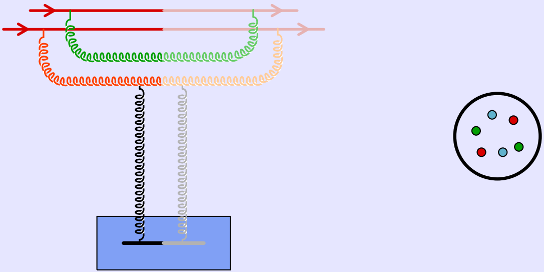


Gluon saturation



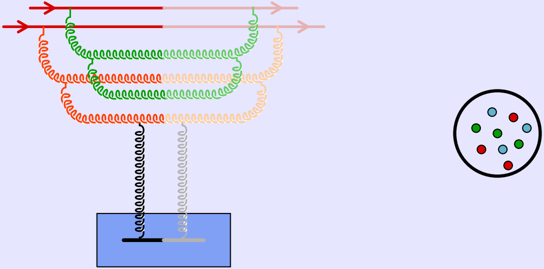
- assume that the projectile is big, e.g. a nucleus, and has many valence quarks (only two are represented)
- on the contrary, consider a small probe, with few partons
- at low energy, only valence quarks are present in the hadron wave function

Gluon saturation



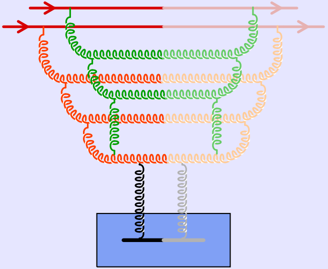
- when energy increases, new partons are emitted
- the emission probability is $\alpha_s \int \frac{dx}{x} \sim \alpha_s \ln(\frac{1}{x})$, with x the longitudinal momentum fraction of the gluon
- at small-x (i.e. high energy), these logs need to be resummed

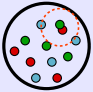
Gluon saturation



- as long as the density of constituents remains small, the evolution is linear: the number of partons produced at a given step is proportional to the number of partons at the previous step (BFKL)

Gluon saturation





- eventually, the partons start overlapping in phase-space
- parton recombination becomes favorable
- after this point, the evolution is non-linear:
  the number of partons created at a given step depends non-linearly on the number of partons present previously

Criterion for gluon recombination

Gribov, Levin, Ryskin (1983)

Number of gluons per unit area :

$$\rho \sim \frac{xG_A(x, Q^2)}{\pi R_A^2}$$

Recombination cross-section :

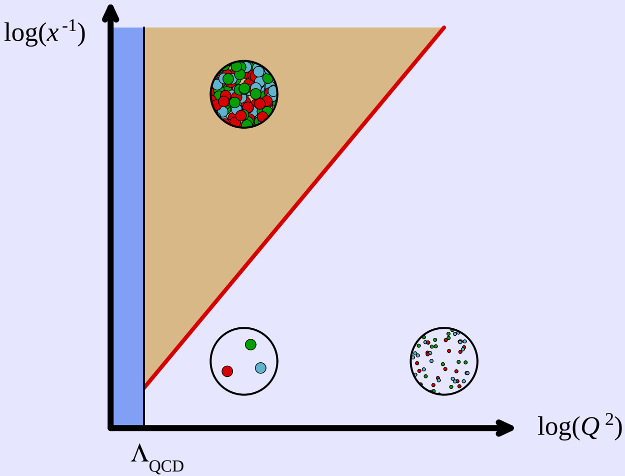
$$\sigma_{gg\to g} \sim \frac{\alpha_s}{Q^2}$$

Recombination happens if $\rho\sigma_{gg\to g} \gtrsim 1$, i.e. $Q^2 \lesssim Q_s^2$, with :

$$Q_s^2 \sim \frac{\alpha_s xG_A(x, Q_s^2)}{\pi R_A^2} \sim A^{1/3}\frac{1}{x^{0.3}}$$

Note: At a given energy, the saturation scale is larger for a nucleus (for $A = 200$, $A^{1/3} \approx 6$)

Saturation domain



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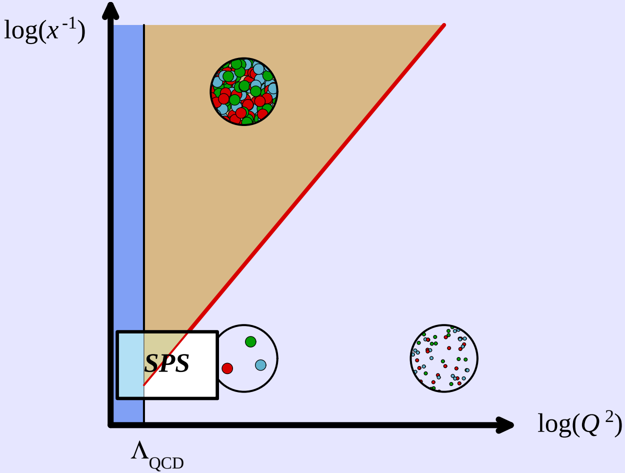

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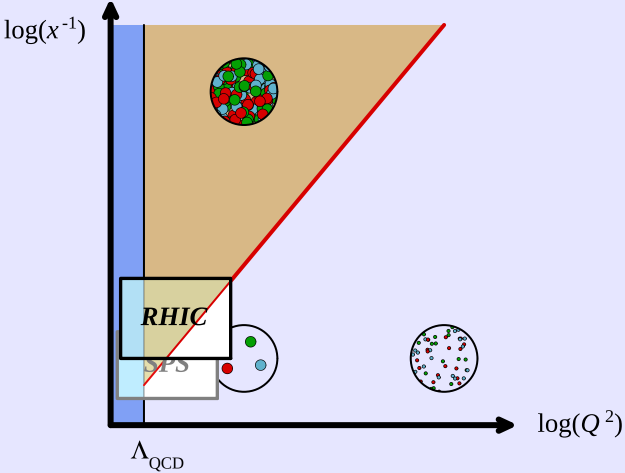

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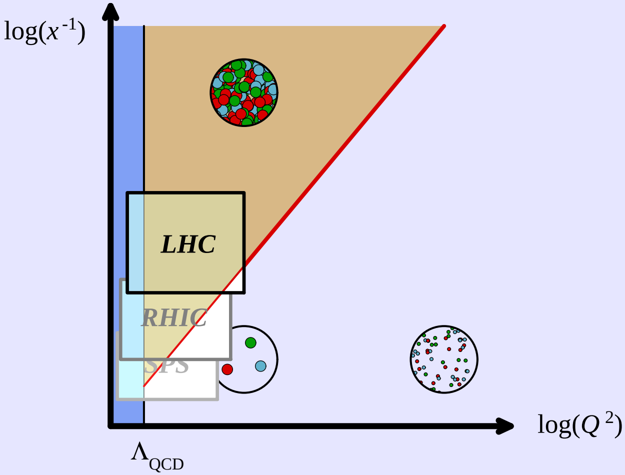

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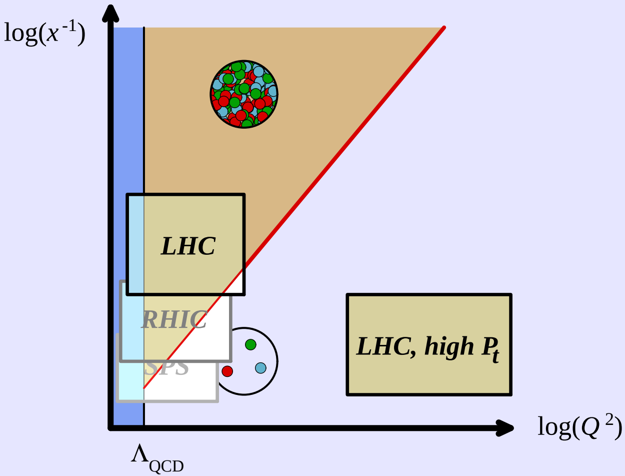


Multiple scatterings

- The saturation criterion can also be seen as a condition for multiple scatterings

- The mean free path of a gluon in a nucleus is

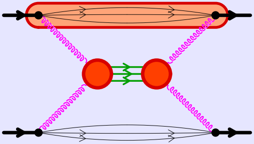
$$\lambda = \frac{1}{n\sigma_{gg\to g}} \quad , \quad n \sim \frac{xG_A(x, Q^2)}{\frac{4}{3}\pi R_A^3}$$

- Multiple scatterings are important if λ becomes smaller than the size of the nucleus, $\lambda \lesssim R_A$, i.e.

$$Q^2 \lesssim \alpha_s \frac{xG_A(x, Q^2)}{\pi R_A^2} \sim Q_s^2$$

Multiple scatterings

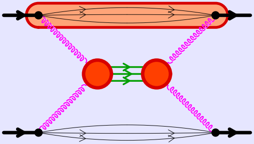
Single scattering :



▷ 2-point function in the projectile ▷ gluon number

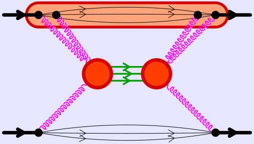
Multiple scatterings

Single scattering :



▷ 2-point function in the projectile ▷ gluon number

Double scattering :



▷ 4-point function in the projectile ▷ higher correlations

Multiple scatterings

- Power counting :

$$\frac{\text{2 scatterings}}{\text{1 scattering}} \sim \frac{Q_s^2}{M_\perp^2} \quad \text{with} \quad Q_s^2 \sim \alpha_s \frac{xG(x,Q_s^2)}{\pi R^2}$$

- When this ratio becomes ~ 1, all the rescattering corrections become important
  - ▷ one must resum all $\left[Q_s/M_\perp\right]^n$
- These effects are not accounted for in DGLAP or BFKL

CGC: Degrees of freedom

CGC = effective theory of small x gluons

- The fast partons (large x) are frozen by time dilation
  ▷ described as static color sources on the light-cone :

$$J^\mu = \delta^{\mu +}\delta(x^-)\rho(\vec{\boldsymbol{x}}_\perp) \qquad\qquad (x^- \equiv (t-z)/\sqrt{2})$$

- Slow partons (small x) cannot be considered static over the time-scales of the collision process
  ▷ they must be treated as standard gauge fields

  Eikonal coupling to the current J^μ : $A_\mu J^\mu$

- The color sources ρ are random, and described by a distribution functional $W_Y[\rho]$, with Y the rapidity that separates “soft” and “hard”

CGC: renormalization group evolution

Evolution equation (JIMWLK) :

$$\frac{\partial W_Y}{\partial Y} = \mathcal{H}\ W_Y$$

$$\mathcal{H} = \frac{1}{2}\int_{\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp} \frac{\delta}{\delta\widetilde{\mathcal{A}}^+(\epsilon,\vec{\boldsymbol{y}}_\perp)}\eta(\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp)\frac{\delta}{\delta\widetilde{\mathcal{A}}^+(\epsilon,\vec{\boldsymbol{x}}_\perp)}$$

where $-\partial_\perp^2\ \widetilde{\mathcal{A}}^+(\epsilon,\vec{\boldsymbol{x}}_\perp) = \rho(\epsilon,\vec{\boldsymbol{x}}_\perp)$

- $\eta(\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp)$ is a non-linear functional of ρ
- This evolution equation resums all the powers of $\alpha_s \ln(1/x)$ and of $Q_s/p_\perp$ that arise in loop corrections
- This equation simplifies into the BFKL equation when the source ρ is small (one can expand η in powers of ρ)

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    - Experimental hints
    - Gluon evolution
    - Saturation domain
    - Multiple scatterings
    - Color Glass Condensate

2. **Deep Inelastic Scattering**
    - Inclusive DIS
    - Exclusive processes

3. **Proton-Nucleus collisions**
    - Link to the dipole cross-section
    - Forward suppression

4. **Nucleus-Nucleus collisions**
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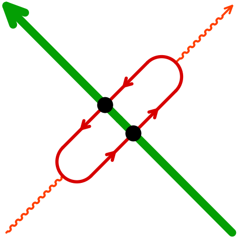
Inclusive DIS

- Reactions involving a hadron or nucleus and an "elementary" projectile are fairly straightforward to study
- The archetype is the forward DIS amplitude :



$$\big\langle \boldsymbol{T}(\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp)\big\rangle = \int [D\rho]\; W_Y[\rho]\;\left[1-\frac{1}{N_c}\mathrm{tr}(U(\vec{\boldsymbol{x}}_\perp)U^\dagger(\vec{\boldsymbol{y}}_\perp))\right]$$

$\triangleright$ this formula resums all the $[\alpha_s \ln(1/x)]^m [Q_s/p_\perp]^n$ for the inclusive DIS cross-section

Inclusive DIS

Inclusive DIS cross-section in the CGC framework

$$\sigma_{\gamma^* T} = \int_0^1 dz \int d^2\vec{\boldsymbol{r}}_\perp \left|\psi(\boldsymbol{q}|z,\vec{\boldsymbol{r}}_\perp)\right|^2 \sigma_{\rm dipole}(x,\vec{\boldsymbol{r}}_\perp)$$

Dipole cross-section

$$\sigma_{\rm dipole}(x,\vec{\boldsymbol{r}}_\perp) \equiv \frac{2}{N_c}\int d^2\vec{\boldsymbol{X}}_\perp \int [D\rho]\, W_x[\rho]$$
$$\times\, {\rm Tr}\Big[1 - \underbrace{U(\vec{\boldsymbol{X}}_\perp + \frac{\vec{\boldsymbol{r}}_\perp}{2})U^\dagger(\vec{\boldsymbol{X}}_\perp - \frac{\vec{\boldsymbol{r}}_\perp}{2})}_{\text{Wilson lines}}\Big]$$

- $|\psi|^2$ is calculable perturbatively in QED
- The distribution $W_x[\rho]$ obeys the JIMWLK equation
- Geometrical scaling: $\sigma_{\rm dipole}(x,r_\perp) \underset{x\to 0}{\to} f(Q_s(x)r_\perp)$

Exclusive processes

Kowalski, Motyka, Watt (2006)

- Inclusive DIS involves only the forward dipole amplitude
- To study more exclusive processes, one needs non-forward amplitudes. They read :

$$\left\langle \Omega_{\rm out} \middle| \gamma^*{}_{\rm in} \right\rangle =$$

$$= \int d^2\vec{\boldsymbol r}_\perp \int_0^1 dz\; \Psi^*_\Omega \psi\; \underbrace{\int d^2\vec{\boldsymbol b}\; e^{i\vec{\boldsymbol q}_\perp\cdot\vec{\boldsymbol b}} \left\langle \boldsymbol T\left(\vec{\boldsymbol b}-\frac{\vec{\boldsymbol r}_\perp}{2}, \vec{\boldsymbol b}+\frac{\vec{\boldsymbol r}_\perp}{2}\right)\right\rangle_Y}_{\substack{\text{non-forward dipole cross-section} \\ \text{with momentum transfer } \vec{\boldsymbol q}_\perp}}$$

Note : this formula assumes that the relevant dipole sizes $r_\perp$ are small compared to the target radius (i.e. the typical $\boldsymbol b$)

Exclusive processes

- By squaring this amplitude, one gets the diffractive cross-section for the production of the state Ω with momentum transfer $\boldsymbol{q}_\perp$

$$\frac{d\sigma^{\text{diff}}_{\gamma^* p\to\Omega p}}{d^2\vec{\boldsymbol{q}}_\perp} = \left|\left<\Omega_{\text{out}}\middle|\gamma^*{}_{\text{in}}\right>\right|^2$$

- The relationship to the inclusive DIS cross-section is

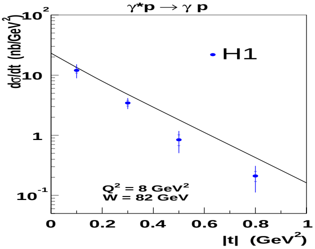
$$\sigma^{\text{tot}}_{\gamma^* p}(Y, Q^2) = 2\,\text{Re}\left<\gamma^*{}_{\text{out}}\middle|\gamma^*{}_{\text{in}}\right>_{\vec{\boldsymbol{q}}_\perp=0}$$

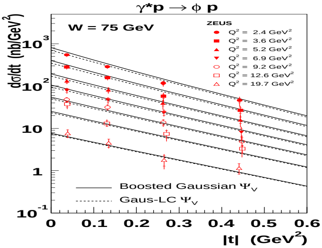
Note : inclusive DIS only constrains the dipole amplitude averaged over impact parameter

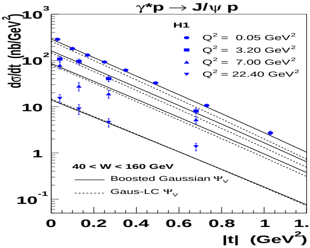
By measuring the $\boldsymbol{q}_\perp$ dependence in exclusive reactions, one obtains informations about the ***b*** dependence of the dipole amplitude

Exclusive reactions

- Exclusive photon and vector meson production :







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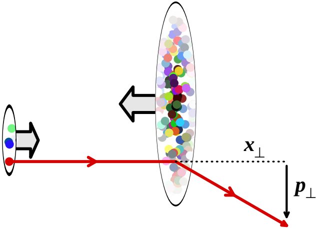
Link to the dipole cross-section

- When the proton is dilute, a pA collision can be seen as a collision between one parton from the proton and the color fields of the nucleus :



- In a given configuration of the target, the scattering amplitude reads :

$$\mathcal{M} \propto \int d^2\vec{\boldsymbol{x}}_\perp \; e^{i\vec{\boldsymbol{p}}_\perp\cdot\vec{\boldsymbol{x}}_\perp} \; U(\boldsymbol{x}_\perp)$$

Note : U is in the representation of the incoming parton

Link to the dipole cross-section

- The squared amplitude, averaged over the target configurations, reads :

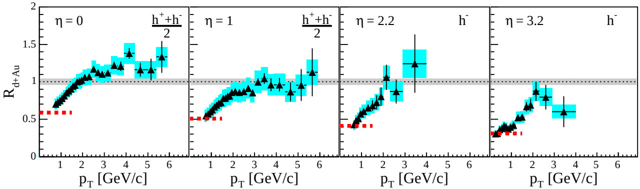
$$
\begin{aligned}
|\mathcal{M}|^2 &\propto \int d^2\vec{\boldsymbol{r}}_\perp\, e^{i\vec{\boldsymbol{p}}_\perp\cdot\vec{\boldsymbol{r}}_\perp} \int d^2\vec{\boldsymbol{b}}\, \left\langle U(\vec{\boldsymbol{b}}+\frac{\vec{\boldsymbol{r}}_\perp}{2})U^\dagger(\vec{\boldsymbol{b}}-\frac{\vec{\boldsymbol{r}}_\perp}{2})\right\rangle_Y \\
&\propto \int d^2\vec{\boldsymbol{r}}_\perp\, e^{i\vec{\boldsymbol{p}}_\perp\cdot\vec{\boldsymbol{r}}_\perp}\, \sigma_{\rm dip}(\vec{\boldsymbol{r}}_\perp, Y)
\end{aligned}
$$

▷ the $p_\perp$-spectrum of the scattered parton is given by the Fourier transform of the dipole cross-section

High pt suppression at large Y

- Results of the BRAHMS experiment at RHIC for deuteron-gold collisions :

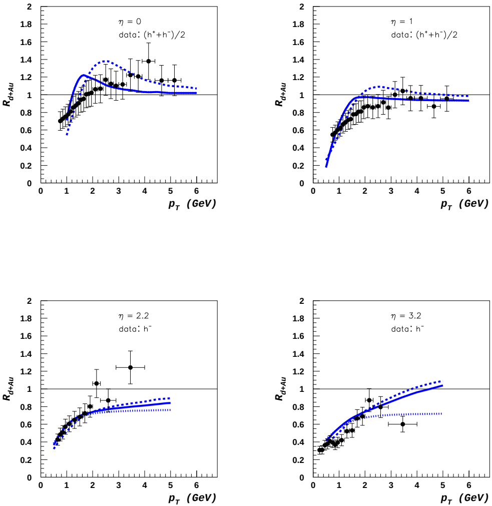
$$R_{\rm dAu} \equiv \frac{1}{N_{\rm coll}} \frac{\left.\frac{dN}{dp_\perp d\eta}\right|_{\rm dAu}}{\left.\frac{dN}{dp_\perp d\eta}\right|_{\rm pp}}$$



- At small rapidity, suppression at low $p_\perp$ and enhancement at high $p_\perp$ (multiple scatterings – Cronin effect)
- At large rapidity, suppression at all $p_\perp$'s (shadowing)

RdA at RHIC from small-x evolution

- Kharzeev, Kovchegov, Tuchin (2005)



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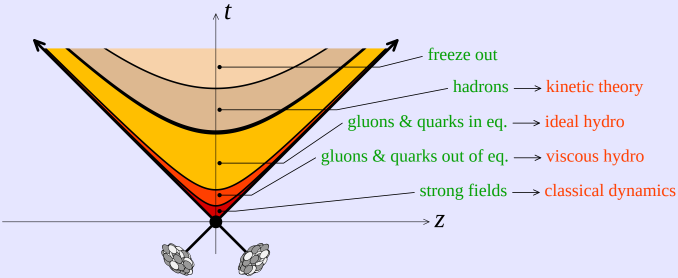
Stages of a nucleus-nucleus collision



- The Color Glass Condensate provides a framework to describe nucleus-nucleus collisions up to a time $\tau \sim Q_s^{-1}$

Reminder on hydrodynamics

Equations of hydrodynamics :

$$\begin{aligned} \partial_\mu T^{\mu\nu} &= 0 \\ \partial_\mu J^\mu_{\rm B} &= 0 \end{aligned}$$

Additional inputs :

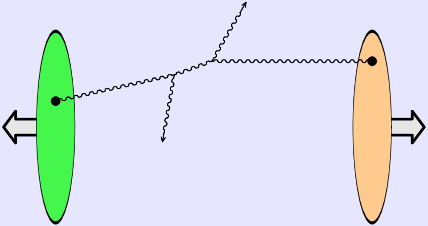
$$p = f(\epsilon) \quad , \quad \eta, \zeta, \cdots$$

- Required initial conditions :

$$T^{\mu\nu}(\tau = \tau_0, \eta, \vec{\boldsymbol{x}}_\perp), J^\mu_{\rm B}(\tau = \tau_0, \eta, \vec{\boldsymbol{x}}_\perp)$$

Initial conditions from CGC: power counting

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{tr}\,F_{\mu\nu}F^{\mu\nu} + (J_1^\mu + J_2^\mu)A_\mu$$



- Dilute regime : one parton in each projectile interact

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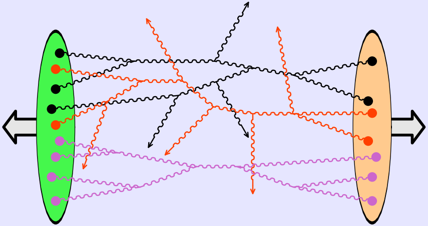


- Dilute regime : one parton in each projectile interact
- Saturated regime : multiparton processes become crucial

  (+ pileup of many partonic scatterings in each AA collision)

Initial conditions from CGC: Leading Order

- In the saturation regime, $\rho_{1,2} \sim g^{-1}$, and we have the following expansion for $T^{\mu\nu}$:

$$T^{\mu\nu} = \frac{Q_s^4}{g^2}\left[c_0 + c_1\, g^2 + c_2\, g^4 + \cdots\right]$$

- The Leading Order contribution is given by classical fields :

$$T^{\mu\nu}_{\rm LO} \equiv c_0 \frac{Q_s^4}{g^2} = \frac{1}{4} g^{\mu\nu}\, \mathcal{F}^{\lambda\sigma}\mathcal{F}_{\lambda\sigma} - \mathcal{F}^{\mu\lambda}\mathcal{F}^{\nu}{}_{\lambda}$$

$$\text{with}\quad \underbrace{\left[\mathcal{D}_\mu, \mathcal{F}^{\mu\nu}\right] = J^\nu}_{\rm Yang-Mills\ \ equation} \quad , \quad \lim_{t\to-\infty} \mathcal{A}^\mu(t,\vec{\boldsymbol{x}}) = 0$$

Initial conditions from CGC: Leading Log resummation

- The previous power counting implicitly assumes that the coefficients c_n are numbers of order one. However, they contain (possibly large) logarithms of $1/x_{1,2}$:

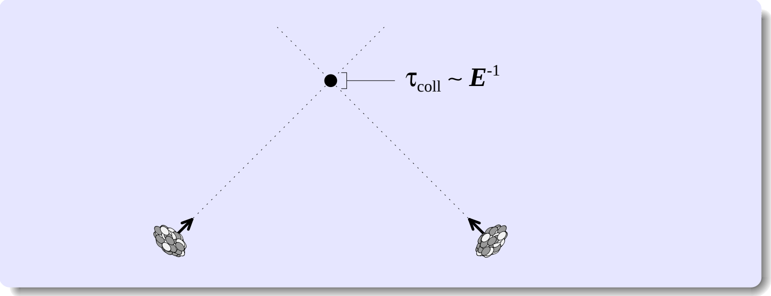
$$\begin{aligned} c_1 &= & & & d_{10} &+ \underbrace{d_{11}\ln\left(\frac{1}{x_{1,2}}\right)} \\ c_2 &= d_{20} &+ d_{21}\ln\left(\frac{1}{x_{1,2}}\right) & & &+ \underbrace{d_{22}\ln^2\left(\frac{1}{x_{1,2}}\right)}_{\text{Leading Log terms}} \end{aligned}$$

$$\left\langle T^{\mu\nu}(\tau,\eta,\vec{\boldsymbol{x}}_\perp)\right\rangle_{\rm LLog} = \int \big[D\rho_1\, D\rho_2\big]\; W_{Y_1}\big[\rho_1\big]\; W_{Y_2}\big[\rho_2\big]\; \underbrace{T^{\mu\nu}_{\rm LO}(\tau,\vec{\boldsymbol{x}}_\perp)}_{\text{for fixed }\rho_{1,2}}$$

with $\partial_Y W = \mathcal{H} W$, $Y_1 = Y_{\rm beam} - \eta$, $Y_2 = Y_{\rm beam} + \eta$

(FG, Lappi, Venugopalan (2008))

Why factorization works: causality



- The duration of the collision is very short: $\tau_{\mathrm{coll}} \sim E^{-1}$

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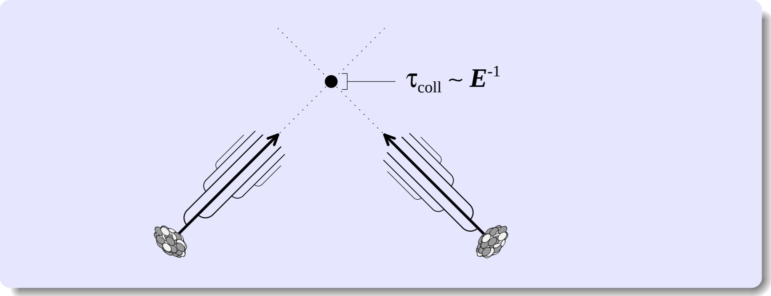

- The duration of the collision is very short: $\tau_{\text{coll}} \sim E^{-1}$
- The logarithms we want to resum arise from the radiation of soft gluons, which takes a long time
  ▷ it must happen (long) before the collision

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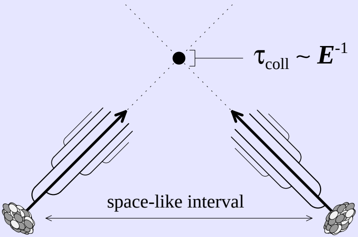

- The duration of the collision is very short: $\tau_{\text{coll}} \sim E^{-1}$
- The logarithms we want to resum arise from the radiation of soft gluons, which takes a long time
  ▷ it must happen (long) before the collision
- The projectiles are not in causal contact before the impact
  ▷ the logarithms are intrinsic properties of the projectiles, independent of the measured observable

Summary

- At a given energy, gluon saturation is enhanced in nuclei
- It plays an important role in the description of the initial stages of nucleus-nucleus collisions
- There are factorization results that relate DIS and AA collisions in the saturated regime
- The $\boldsymbol{p}_\perp$-dependence of single inclusive spectra in p(d)-A collisions gives the dipole size dependence of the dipole cross-section
- The t-dependence of exclusive processes gives its impact parameter dependence